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Tensile-shear transition in mixed mode I/III fracture

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Abstract

The propensity of the transition of fracture type in either brittle or ductile cracked solid under mixed-mode I and III loading conditions is investigated. A fracture criterion based on the competition of the maximum normal stress and maximum shear stress is utilized. The prediction of the fracture type is determined by comparing $\tau_{\max}/\sigma_{\max}$ at a critical distance from the crack tip to the material strength ratio τ_C/σ_C , i.e., $(\tau_{\max}/\sigma_{\max}) < (\tau_C/\sigma_C)$ for tensile fracture and $(\tau_{\max}/\sigma_{\max}) > (\tau_C/\sigma_C)$ for shear fracture, where σ_C (τ_C) is the fracture strength of materials in tension (shear). Mixed mode I/III fracture tests were performed using circumferentially notched cylindrical bars made of PMMA and 7050 aluminum alloy. Fracture surface morphology of the specimens reveals that: (1) for the brittle material, PMMA, only tensile type of fracture occurs, and (2) for the ductile material, 7050 aluminum alloy, either tensile or shear type of fracture occurs depending on the mode mixity. The transition (in ductile material) or non-transition (in brittle material) of the fracture type and the fracture path observed in experiments were properly predicted by the theory. Additional test data from open literature are also included to validate the proposed theory.

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1. Introduction

Fracture of brittle and ductile solids under mixed mode loading conditions has been of special interests in both experiments and theoretical studies due to its close proximity to practical loading conditions of engineering structures. Fracture criteria are needed for accurate prediction of fracture type and transition, crack initiation/growth angle and path, and the onset fracture load.

The earliest and also the most popular fracture criterion for mixed mode I/II fracture of *brittle materials* was made by Erdogan and Sih (1963) who proposed the maximum hoop stress criterion (MHSC). The criterion says: (1) the fracture is governed by the attainment of a critical hoop stress, using a polar coordinate system centered at the crack tip, over a characteristic distance around the crack tip, and (2) the fracture starts at its main crack tip and is in a direction perpendicular to the critical hoop stress direction.

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For mixed mode I/II fracture of *ductile materials* which appears to have mode II dominated fracture, Maccagno and Knott (1992) initially proposed the maximum shear stress criterion (MSSC), and it says: (1) fracture initiates from the crack tip in the radial direction, (2) fracture is governed by the achievement of a critical shear stress, using a polar coordinate system centralized at the crack tip, over a characteristic length around the crack tip, and (3) the incipient crack growth is along the critical shear stress direction. In the papers by Chao and Liu (1997) and Chao and Zhu (1999) for mixed mode I/II, the authors extended the basic ideas of MHSC and MSSC by assuming the same characteristic distance around the crack tip for both hoop and shear stress, and incorporated the concept of material strength ratio with MHSC and MSSC into a new unified failure criterion for the prediction of fracture transition from tensile to shear type. The proposed theory is able to demonstrate: (1) why brittle materials always fail in tensile type regardless of the mode mixity, and (2) under what mode-mixity the transition of fracture type would occur in ductile materials, and (3) the fracture path which accords to the fracture type. In the present paper, the previous theory is further extended to mixed mode I/III fracture. A brief literature survey of the past work for mixed mode I/III is first given in this section, followed by theoretical development, fracture tests and finally comparison of the theory with test data.

There is considerable work done on the mixed mode I/III fracture in *brittle material* regimes. It appears that the first experimental study of mixed mode I/III fracture was made by Sommer (1969) who used circumferentially notched glass cylinders. He found that the fracture surface morphology was characterized by radial, three-dimensional macro-structures ('lances') that were regularly spaced around the crack front. Similar morphology, termed "factory roof", encountered in testing of ceramics and other *brittle* materials, was observed by other researchers (Knauss, 1970; Lazarus and Leblond, 1998; Chai, 1988; Suresh and Tscheegg, 1987; Tscheegg and Suresh, 1988; Suresh et al., 1990; Hsia et al., 1995). The failure mechanism in mixed mode I/III brittle fracture is found only in *tensile type (mode I dominated)*. In addition, compared with mixed mode I/II, the mixed mode I/III is more cumbersome and sophisticated because the superposition of the mode I/III loading results in multiple fracture planes intersecting the crack front which allows for easy stress relief by way of mode I cracking. Consequently, the failure surface in mixed mode I/III fracture is typically non-planar.

Several fracture criteria have been proposed to date for the mixed mode I/III fracture of solids, e.g., (a) minimum strain energy density criterion (Sih and Cha, 1974; Sih and Barthelemy, 1980; Chen et al., 1986), which postulates that the crack will propagate along the plane with a minimum strain energy density at a characteristic length, (b) maximum normal stress criterion (MNSC) (Tian et al., 1982; Yates and Miller, 1989), which suggests that the fracture is governed by the attainment of a critical normal stress and the crack will then propagate along the plane normal to the maximum normal stress, (c) maximum principal stress criterion (Yates and Miller, 1989), which claims that the crack propagates in a direction perpendicular to the maximum principal stress, and (d) maximum strain energy release rate criterion (Pook and Sharples, 1979, 1985; Chen et al., 1986; Yates and Miller, 1989), which postulates that the crack under mixed-mode loading will propagate along a direction which maximizes the energy release rate.

It is found that the prediction of both direction of crack growth and fracture loads from the above four fracture criteria differs only slightly. Furthermore, since the fracture surface is quite "irregular", it is rather laborious to measure the initiation angles from the fracture surface. As such, experimental data on crack initiation angles under monotonic loading conditions are not readily available, although there are limited literatures reporting crack initiation angles in fatigue (Pook, 1985; Yates and Miller, 1989).

Test data of brittle materials show that the critical stress intensity factor at fracture initiation increases as the loading mode is changed from pure tension to pure torsion. This is often attributed to several reasons. Firstly, the friction between the crack surfaces (Suresh and Tscheegg, 1987; Pook, 1985; Hsia et al., 1995) may contribute to the increase of the fracture resistance. Secondly, the fracture surfaces in the mixed mode I/III case exhibit rough appearances such that the actual fracture area is larger than the assumed planar surface (Manoharan et al., 1991) in most of the theoretical analyses. The increase of the fracture surface

area may elevate the apparent fracture toughness. Thirdly, Mode III fracture behavior differs phenomenologically from both mode I and mode II fracture (Suresh and Tschegg, 1987; Tschegg and Suresh, 1988; Suresh et al., 1990). During fracture, the first suitably oriented microcrack is a precursor to sample failure. Microcracking is much more prevalent for mode III fracture than that for mode I or II fracture. That is, more microcracks contributing to brittle fracture could link up and lead to the cleavage or inter-granular fracture of the whole specimen. The linking up of many of the microcracks induces the increase of the fracture toughness.

For materials with certain amount of ductility at failure, similar to mixed-mode I/II, the mixed-mode I/III ductile fracture is complicated by two distinctive macroscopic fracture mechanisms, i.e., tensile type and shear type, as observed from fractographs. And the failure type of a given specimen depends on the loading mixity. Shah (1974) observed that for 4340 steel, the specimen (round-notched bar) subjected to pure tension had a flat fracture surface, while specimens under mixed mode loading with K_I/K_{III} ratios of 2.30 and 1.16 exhibited a non-flat fracture surface. However, since the fracture surfaces in $K_I/K_{III} = 2.30$ and 1.16 cases revealed the same texture as those of the pure tension fracture, he then concluded that mode I (K_I) played the predominant role in the fracture of these specimens. On the other hand, the texture of fracture surface of the specimen of $K_I/K_{III} = 0.63$ was alike that of the fracture under pure torsion, indicating mode III (K_{III}) played a dominant role in fracture. The fracture surface of the specimen subjected to pure torsion was flat with shear rubbing marks. Similar test results of fracture type transition have been reported for 7075-T651 aluminum alloy (Williams and Ewing, 1972), 2034 aluminum alloy (Feng et al., 1993; Kamat and Hirth, 1996), F-82H stainless steel (Kamat and Hirth, 1996), and 2024-T3 aluminum alloy (Helm et al., 2001; Sutton et al., 2001).

In addition to using stress intensity factors K_I and K_{III} as the fracture parameters, other fracture parameters were also used to describe the mixed mode ductile fracture, e.g., the modified J integrals denoted as J_{IC} and J_{IIC} to characterize the elastic–plastic fracture (Kamat and Hirth, 1994, 1996; Manoharan, 1995), damage mechanics parameters by Gao and Shih (1998), the crack-tip displacement CTD_{III} and the plastic strain intensity Γ_{III} for pure mode III fracture (Tschegg and Suresh, 1988; Ritchie et al., 1985). However, it appears that all these fracture parameters and theories cannot be used to predict the transition of failure from tensile type to shear type.

In this paper, we present a novel fracture criterion for the interpretation of the transition in fracture type in mixed mode I/III for both brittle and ductile engineering materials. This theory is an extension of

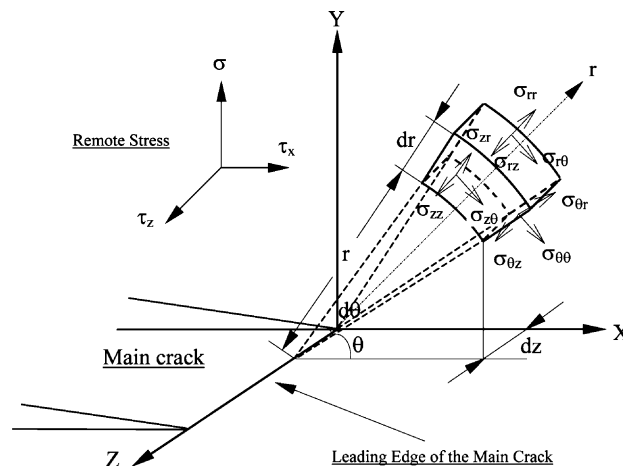


Fig. 1. Mixed mode I/II/III stress fields near the crack tip.

Erdogan and Sih (1963), Maccagno and Knott (1992), Chao and Liu (1997) and Chao and Zhu (1999) and based on the competition of maximum normal/shear stress at the same characteristic distance from the crack tip. Only linear elastic fracture mechanics (LEFM) solutions are applied to predict the load and crack propagation direction. Material strength ratio is reflected by τ_c/σ_c and the failure of ductile materials is assumed to obey either the classical Tresca or von Mises failure theory. Experiments are then described and the results are reported to validate the proposed theory. Tests are carried out for PMMA and 7050 aluminum alloy using a circumferential fatigue pre-cracked round bar under combined remote tension and torsion load. The PMMA is chosen as the representative brittle material, and 7050 aluminum alloy an engineering ductile material. The fractographs from broken samples are examined in order to correlate the fracture mechanisms with the observed macroscopic failure types. The fracture initiation loads and angles are measured and compared with the proposed fracture criterion. In addition, test data from Suresh and Tschegg (1987), Pook (1985), and Shah (1974) are also used to compare with the fracture criterion. Note that the objective of the work is to predict the transition of fracture type in either brittle or ductile materials with less emphasis on the fracture load.

2. Theoretical consideration

For the mode I/II case, the crack tip stress fields for either the linear elastic or elastic–plastic case are available. However, complete solutions for the elastic–plastic case of mode I/III do not exist. The partial results provided by Pan and Shih (1990, 1992) for mode I/III cases are not sufficient for our analysis. Nevertheless, our previous analysis of *failure transitions and initiation angle* for the mode I/II indicates only a very small difference between the elastic and elastic–plastic crack tip solutions (Chao and Liu, 1997). Therefore, only linear-elastic crack tip fields will be discussed and used for the prediction of the fracture initiation angle and the transition of fracture mode in the current work.

2.1. Mixed mode I/III crack tip stress fields

Using the cylindrical coordinates (r, θ, z) as shown in Fig. 3 and within the context of linear elastic and homogeneous solids, the singular crack tip stress fields under mixed mode I/III conditions can be written as:

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left(1 + \sin^2 \frac{\theta}{2}\right) \quad (1)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \cos^3\left(\frac{\theta}{2}\right) \quad (2)$$

$$\sigma_{r\theta} = \frac{K_I}{2\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \sin \theta \quad (3)$$

$$\sigma_{zz} = \begin{cases} 0 & \text{(Plane stress)} \\ \nu(\sigma_{\theta\theta} + \sigma_{rr}) & \text{(Plane strain)} \end{cases} \quad (4)$$

$$\sigma_{rz} = \frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \quad (5)$$

$$\sigma_{\theta z} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \quad (6)$$

Enlightened by the fracture morphology of mixed mode I and III fracture and by the hypotheses in MHSC from Erdogan and Sih (1963), MSSC from Maccagno and Knott (1992), and the combined MHSC and MSSC from Chao and Liu (1997) and Chao and Zhu (1999), it is assumed that for the fracture of elastic and elastic–plastic solids under mixed mode I and III loading conditions,

- (1) crack plane(s) start at its main crack tip in the radial direction; and
- (2) crack plane(s) start in the directions either perpendicular to the maximum tensile stress or parallel to the maximum shear stress.

Following the above two assumptions, the maximum normal (shear) stress on the $\theta - Z$ plane, *which is in the hoop (radial) direction* as shown in Figs. 1 and 4, dictates or triggers the tensile (shear) type of fracture of material. In order to obtain the maximum stresses, coordinate transformation between the two coordinate systems (r, θ, Z) and (r', α, Z') was applied as shown in Fig. 2. The normal stress and shear stress in the new coordinate system (r', α, Z') are given by:

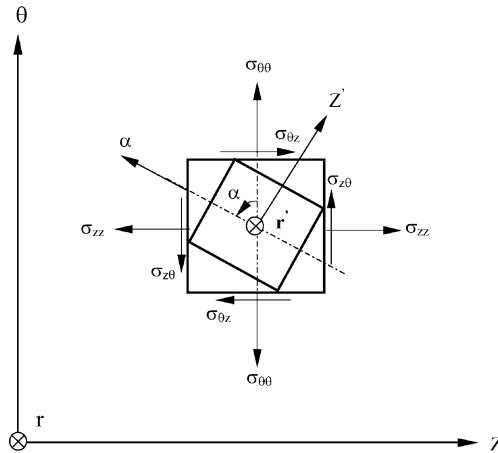


Fig. 2. Normal and shear stresses in $\theta - Z$ plane.

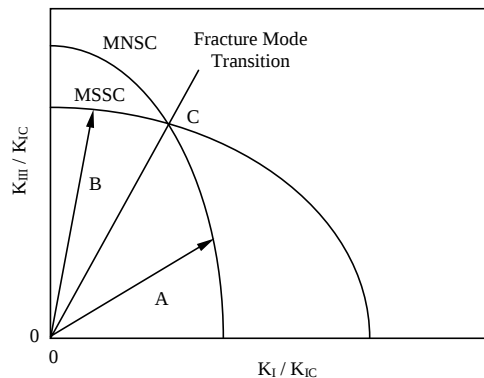


Fig. 3. Schematics showing the competition of MNSC and MSSC.

$$\sigma_{xx} = \sigma(\theta, \alpha) = \frac{K_I}{\sqrt{2\pi r}} \cos^3 \frac{\theta}{2} \cos^2 \alpha - \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin 2\alpha + \frac{2\nu K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin^2 \alpha \quad (7)$$

$$\tau_{xz'} = \tau(\theta, \alpha) = \frac{K_I}{2\sqrt{2\pi r}} \cos^3 \frac{\theta}{2} \sin 2\alpha + \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \cos 2\alpha - \frac{\nu K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin 2\alpha \quad (8)$$

Note that in Eqs. (7) and (8), the plane strain condition, $\sigma_{zz} = \nu(\sigma_{\theta\theta} + \sigma_{rr})$, is used. The solutions for the plane stress condition, $\sigma_{zz} = 0$, can be obtained from the plane strain case by setting $\nu = 0$.

Using (7) and (8), we can then derive the maximum normal stress and maximum shear stress in the $\theta - Z$ plane, and establish the fracture criteria, i.e., maximum normal stress criterion (MNSC) and maximum shear stress criterion (MSSC), respectively, for the mixed mode I/III loading condition.

2.2. Maximum normal stress criterion (MNSC)

From Eq. (7), the *maximum* normal stress at a specific r occurs at the angle θ^* ($-\pi \leq \theta^* \leq \pi$) and α^* ($-\frac{\pi}{2} \leq \alpha^* \leq \frac{\pi}{2}$) which satisfy

$$\frac{\partial \sigma_{xx}}{\partial \theta} = -\frac{1}{2} \sin \frac{\theta}{2} \left(\frac{3K_I}{\sqrt{2\pi r}} \cos^2 \frac{\theta}{2} \cos^2 \alpha - \frac{K_{III}}{\sqrt{2\pi r}} \sin 2\alpha + \frac{2\nu K_I}{\sqrt{2\pi r}} \sin^2 \alpha \right) = 0 \quad (9)$$

and

$$\frac{\partial \sigma_{xx}}{\partial \alpha} = \cos \frac{\theta}{2} \left[\frac{K_I}{\sqrt{2\pi r}} \left(2\nu - \cos^2 \frac{\theta}{2} \right) \sin 2\alpha - \frac{2K_{III}}{\sqrt{2\pi r}} \cos 2\alpha \right] = 0 \quad (10)$$

Solving Eq. (9) yields

$$\theta = 0 \quad (11)$$

or

$$\frac{3K_I}{\sqrt{2\pi r}} \cos^2 \frac{\theta}{2} \cos^2 \alpha - \frac{K_{III}}{\sqrt{2\pi r}} \sin 2\alpha + \frac{2\nu K_I}{\sqrt{2\pi r}} \sin^2 \alpha = 0 \quad (12)$$

Solving Eq. (10) yields

$$\theta = \pm \pi \quad (13)$$

or

$$\frac{K_I}{\sqrt{2\pi r}} \left(2\nu - \cos^2 \frac{\theta}{2} \right) \sin 2\alpha - \frac{2K_{III}}{\sqrt{2\pi r}} \cos 2\alpha = 0 \quad (14)$$

Eqs. (11)–(14) show that the possible extreme values of σ_{xx} could exist at the locations (θ, α) governed by (11) and (14), or (12) and (13), or (12) and (14). Using the mathematical theorem of extreme values of two-variables (Liu, 2002), it is found that the maximum normal stress σ_{xx} at r occurs at

$$\theta^* = 0 \quad (15)$$

and

$$\alpha^* = \frac{1}{2} \tan^{-1} \left[\frac{2}{(2\nu - 1)D_{13}} \right] \quad (16)$$

$$\begin{cases} -\frac{\pi}{4} \leq \alpha^* \leq 0 & (\text{for } K_{III} \geq 0) \\ 0 \leq \alpha^* \leq \frac{\pi}{4} & (\text{for } K_{III} \leq 0) \end{cases} \quad (17)$$

where $D_{13} = K_I/K_{III}$. Substituting (15) and (16) into (7) one obtains the maximum normal stress σ_{xx} as

$$\sigma_{xx}(\theta^*, \alpha^*) = \sigma_{xx}(0, \alpha^*) = \frac{K_I}{\sqrt{2\pi r}} \cos^2 \alpha^* - \frac{K_{III}}{\sqrt{2\pi r}} \sin 2\alpha^* + \frac{2\nu K_I}{\sqrt{2\pi r}} \sin^2 \alpha^* \quad (18)$$

From (18) and applying the concept of fracture toughness, we obtain the material failure envelope in the K_I – K_{III} space as

$$\frac{K_I}{K_{IC}} \cos^2 \alpha^* - \frac{K_{III}}{K_{IC}} \sin 2\alpha^* + \frac{2\nu K_I}{K_{IC}} \sin^2 \alpha^* = 1 \quad (19)$$

where $K_{IC} = \sigma_C \sqrt{2\pi r_C}$; K_{IC} is the fracture toughness under pure mode I conditions, σ_C is the critical stress, and r_C is the characteristic or critical distance. The symbol K_{IC} used here is the K_I at the fracture of the specimen under pure mode I conditions, and could be different from the plane strain fracture toughness defined in the ASTM testing standards.

It is understood that the fracture criterion, “the attainment of a critical stress at a critical distance” used by Erdogan and Sih (1963) for mixed mode I/II case, is also implied here for mixed mode I/III case. Furthermore, based on MNSC, when $\theta^* = 0$ and $\alpha^* = 0^\circ$, Eq. (19) gives $K_I = K_{IC}$ for pure mode I conditions; while as $\theta^* = 0$ and $\alpha^* = -45^\circ$ ($+45^\circ$), it gives $K_{III} = K_{IIIc} = K_{IC}$ ($K_{III} = K_{IIIc} = -K_{IC}$) for pure mode III conditions.

Note that Eqs. (15) and (16) imply that the crack would propagate macroscopically in the same direction as that of the crack face, i.e., $\theta^* = 0$, and with the “factory roof” type of fracture surface. Because $\sigma_{\theta\theta}|_{\theta=0}$ (in the Y direction in Fig. 1) is perpendicular to the plane of the main crack, and $\sigma_{xx}(\theta = 0, \alpha^*)$ is normal to the plane of fracture “roof”, then the angle from the plane of main crack to the plane of fracture “roof” is equivalent to the angle from $\sigma_{\theta\theta}|_{\theta=0}$ to $\sigma_{xx}(\theta = 0, \alpha^*)$, i.e., the fracture angle is α^* , and it is a function of the applied mode mixity and Poisson’s ratio ν .

2.3. Maximum shear stress criterion (MSSC)

Since the maximum shear stress occurs in a plane that is $\pi/4$ from that of the maximum normal stress and using θ^{**} ($-\pi \leq \theta^{**} \leq \pi$) and α^{**} ($-\frac{\pi}{2} \leq \alpha^{**} \leq \frac{\pi}{2}$) to specify the plane on which the maximum shear stress at r occurs, it gives (see Liu (2002) for more detailed discussions)

$$\theta^{**} = 0 \quad (20)$$

$$\alpha^{**} - \alpha^* = \frac{\pi}{4} \quad (21)$$

Substituting Eq. (21) into Eq. (16), one arrives at

$$\alpha^{**} = \frac{1}{2} \tan^{-1} \left[\frac{D_{13}(1-2\nu)}{2} \right] \quad (22)$$

and

$$\begin{cases} 0 \leq \alpha^{**} \leq \frac{\pi}{4} & (\text{for } K_{III} \geq 0) \\ -\frac{\pi}{4} \leq \alpha^{**} \leq 0 & (\text{for } K_{III} \leq 0) \end{cases} \quad (23)$$

where $D_{13} = K_I/K_{III}$.

Substituting (20) and (22) into (8), one has the maximum shear stress $\tau_{\alpha\alpha'}$

$$\tau_{\alpha\alpha'}(\theta^{**}, \alpha^{**}) = \tau_{\alpha\alpha'}(0, \alpha^{**}) = \frac{(\frac{1}{2} - \nu)K_I}{\sqrt{2\pi r}} \sin 2\alpha^{**} + \frac{K_{III}}{\sqrt{2\pi r}} \cos 2\alpha^{**} \quad (24)$$

If the maximum shear stress $\tau_{zz'}$ reaches the critical shear stress τ_c , which is defined as the fracture stress of the material in pure shear, then

$$\tau_c = \frac{(\frac{1}{2} - \nu)K_I}{\sqrt{2\pi r_c}} \sin 2\alpha^{**} + \frac{K_{III}}{\sqrt{2\pi r_c}} \cos 2\alpha^{**} \quad (25)$$

Applying the concept of fracture toughness, i.e., $K_{IC} = \sigma_c \sqrt{2\pi r_c}$ we therefore have

$$\tau_c = \tau_c \frac{K_{IC}}{\sigma_c \sqrt{2\pi r_c}} \quad (26)$$

Substituting Eq. (26) into Eq. (25) and simplifying, we obtain the material failure envelope in the K_I and K_{III} space as

$$\frac{\sigma_c}{\tau_c} \left(\frac{K_I}{2K_{IC}} \sin 2\alpha^{**} + \frac{K_{III}}{K_{IC}} \cos 2\alpha^{**} - \frac{\nu K_I}{K_{IC}} \sin 2\alpha^{**} \right) = 1 \quad (27)$$

The failure envelope, as shown by Eq. (27), is a function of the material strength ratio, defined here as τ_c/σ_c . Based on MSSC for pure mode I conditions, when $\theta^* = 0$ and $\alpha^{**} = 45^\circ$, Eq. (27) predicts $K_I = \frac{2}{1-2\nu} \frac{\tau_c}{\sigma_c} K_{IC}$. In addition, based on MSSC for pure mode III conditions, when $\theta^* = 0$ and $\alpha^{**} = 0$, it gives $K_{III} = K_{IIIc} = \tau_c/\sigma_c K_{IC}$.

Note that similar to Eqs. (15) and (16) in MNSC, Eqs. (20)–(22) also imply that the crack would grow macroscopically in the identical direction as that of the crack face, i.e., $\theta^{**} = 0$, and with the “factory roof” type of fracture surface. Because $\sigma_{\theta\theta}|_{\theta=0}$ (in the Y direction in Fig. 1) is perpendicular to the plane of main crack, and the angle from $\sigma_{zz}(\theta = 0, \alpha^{**})$ to $\tau_{zz'}(\theta = 0, \alpha^{**})$ is α^{**} (in counterclockwise direction), then the angle from $\sigma_{\theta\theta}|_{\theta=0}$ to the plane of “roof”, which is parallel to $\tau_{zz'}(\theta = 0, \alpha^{**})$ and radius r , is also α^{**} . Therefore, the fracture angle, i.e., the angle from the plane of main crack to the plane of “roof”, in counterclockwise direction, is $\alpha^{**} + \pi/2$.

2.4. Competition between MNSC and MSSC

It is assumed that a material can fail by either tensile or shear, depending on which stress reaches its corresponding critical value first. When this criterion is applied to the crack problem, it can be stated and assumed as, by evaluating $(\sigma_{zz})_{\max}$ and $(\tau_{zz'})_{\max}$ at the same critical distance/radius $r = r_c$ near the crack tip of a mixed-mode loaded specimen or structure, fracture commences when either $(\sigma_{zz})_{\max} \geq \sigma_c$ obeying MNSC, or when $(\tau_{zz'})_{\max} \geq \tau_c$ obeying MSSC. The type of the fracture, i.e., MNSC for tensile type and MSSC for shear type, depends on which is satisfied first as the load level is increased. As shown schematically in Fig. 3, the failure loci based on the MNSC, i.e., Eq. (19), and the MSSC, i.e., Eq. (27) is presented. A specimen with loading path A (B), shown in Fig. 3, will fail according to the MNSC (MSSC). The transition of the type of fracture from MNSC (tensile type) to MSSC (shear type) would occur at the intersection point C. Using Eqs. (19) and (27), the intersection point C is determined as

$$\frac{K_I}{K_{III}} = \frac{\cos 2\alpha^{**} + \frac{\tau_c}{\sigma_c} \sin 2\alpha^{**}}{\frac{\tau_c}{\sigma_c} \cos^2 \alpha^{**} + \frac{2\nu\tau_c}{\sigma_c} \sin^2 \alpha^{**} - \frac{1}{2} \sin 2\alpha^{**} + \nu \sin 2\alpha^{**}} \quad (28)$$

Eq. (28) indicates that the fracture transition is a function of the mode mixity K_I/K_{III} , Poisson's ratio ν , and the material strength ratio τ_c/σ_c . It is solved numerically and plotted in Fig. 4 (using $\nu = 1/3$ for plane strain,) and Fig. 5 ($\nu = 0$ in Eq. (19) for plane stress).

As shown in Fig. 4, the transition from tensile type to shear type of fracture for a pre-cracked solid can be determined for a given material, represented by τ_c/σ_c , and the loading condition, K_I/K_{III} . If a combination of τ_c/σ_c and K_I/K_{III} falls on the right (left) side of the curve, plotted with vertical (slant) lines,

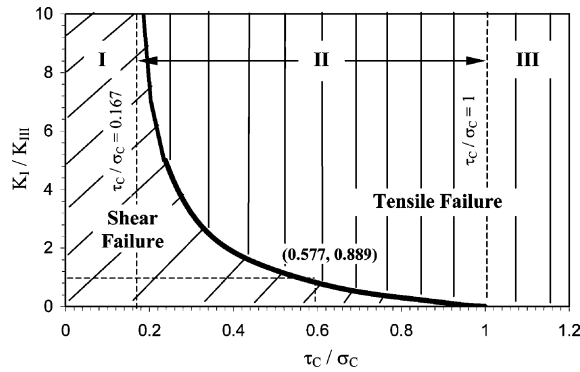


Fig. 4. Fracture mode as determined by the loading condition (K_I/K_{III}) and the material strength ratio (τ_C/σ_C) under plane strain conditions ($\nu = 1/3$).

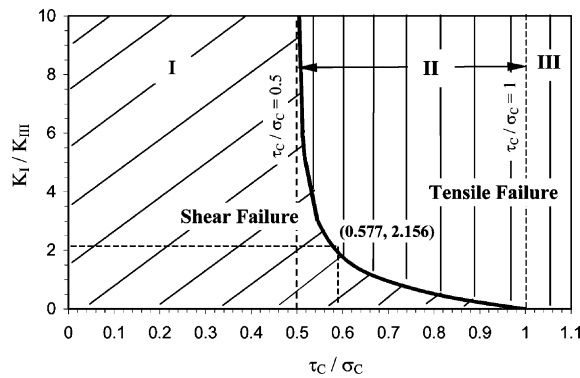


Fig. 5. Fracture mode as determined by the loading condition (K_I/K_{III}) and the material strength ratio (τ_C/σ_C) under plane stress conditions.

tensile (shear) fracture would occur. The transition approaches infinity as $\tau_C/\sigma_C = 0.167$, and is zero at $\tau_C/\sigma_C = 1$. Fig. 4 demonstrates that there exist three zones. In Zone I, for materials with $\tau_C/\sigma_C < 0.167$ which represent very ductile materials, only shear failure is possible regardless of the mode mixity. In Zone II, for materials with $0.167 < \tau_C/\sigma_C < 1$, either shear/tensile failure can occur, depending on the mode mixity. In Zone III, for materials with $\tau_C/\sigma_C > 1$ which represent very brittle materials, only tensile failure is possible regardless of the mode mixity.

As shown in Fig. 5, the transition behavior under plane stress case is similar to that of the plane strain case. The only difference of the two cases is that the transition point between Zone I and Zone II is at $\tau_C/\sigma_C = 0.167$ in Fig. 4 and 0.5 in Fig. 5.

2.5. Material fracture strength ratio

Note that the critical tensile stress σ_C and critical shear stress τ_C are used in the current discussion as the material fracture strength of flawed specimens. These two properties can be assumed to be proportional to the uniaxial tensile fracture strength and pure torsional fracture strength of smooth specimens, respectively. In other words, the material fracture strength ratio τ_C/σ_C is assumed to be equal to the material fracture strength ratio of smooth specimens. Therefore, σ_C and τ_C can be related to each other by the classical failure

theories. For instance, the maximum normal stress theory is a well-accepted failure criterion for highly brittle materials, and this failure theory gives $\tau_C = \sigma_C$ (Mendelson, 1968). For ductile materials, the maximum shear stress (Tresca) theory gives $\tau_C = 0.5 \sigma_C$, and the distortion energy or von Mises theory gives $\tau_C = \frac{\sigma_C}{\sqrt{3}} = 0.577 \sigma_C$ (Mendelson, 1968). Furthermore, test results have shown that the Tresca theory is generally a lower bound for ductile materials. Thus, the material strength ratio, τ_C/σ_C , ranges from 0.5 for highly ductile to unity for highly brittle materials. Structural materials would generally have a τ_C/σ_C value between 0.5 and 1, based on these failure theories.

Most materials handbooks do not provide the fracture strength of the materials of smooth specimens. However, ultimate strengths of smooth specimens in tension and shear are sometimes given. For instance, the ultimate strength in tension and shear are given as 550 MPa and 325 MPa, respectively, for the aluminum 7050 (Aerospace Structural Metals Handbook, 1993), and 1896 MPa (Shah, 1974) and 1207 MPa for the 4340 steel (Aerospace Structural Metals Handbook, 1993). If it is assumed that the material strength ratio can be approximated by the ultimate strength ratio, the materials strength ratio is, therefore, $(\tau_C/\sigma_C) = (325/550) = 0.590$ for the aluminum alloy 7050, and $(\tau_C/\sigma_C) = (1207/1896) = 0.637$ for the 4340 steel. These values are close to 0.577 as determined by the von Mises failure theory.

Using the curve for fracture transition in Fig. 4, $K_I/K_{III} = 0.889$ as $\tau_C/\sigma_C = 0.577$ if the material obeys the von Mises failure theory. Therefore, when loading condition K_I/K_{III} is larger (smaller) than 0.889 in plane strain conditions, the curve in Fig. 4 or Eq. (26) predicts that ductile materials would fail in tension (shear) mode. Under the plane stress conditions, the critical mode mixity K_I/K_{III} is 2.156 as $\tau_C/\sigma_C = 0.577$ as shown in Fig. 5.

Using the competition of MNSC and MSSC as the fracture criterion, Fig. 6 shows schematically the effect of material strength ratio and loading path on the prediction of the type of fracture. Note that the loading path or the stress state, specified by $(\tau_{xz'})_{\max}/(\sigma_{xx})_{\max}$ at the crack tip, is linear in the elastic range and may be approximated as linear in the plastic range if the proportional loading conditions holds. As shown in Fig. 6, tensile type of fracture would commence if the loading path for a given specimen or structure is below the material strength ratio line, i.e., $[(\tau_{xz'})_{\max}/(\sigma_{xx})_{\max}] < (\tau_C/\sigma_C)$, since the loading path or $(\sigma_{xx})_{\max}$ would reach σ_C first as the load is increased. On the other hand, shear type of fracture would commence if the loading path for a given specimen or structure is above the material strength ratio line, i.e., $[(\tau_{xz'})_{\max}/(\sigma_{xx})_{\max}] > (\tau_C/\sigma_C)$, since the loading path or $(\tau_{xz'})_{\max}$ would reach τ_C first as the load is increased. It should be noted that although Figs. 5 and 8 present differently the underlying theory is the same since point C in Fig. 3 corresponds to $[(\tau_{xz'})_{\max}/(\sigma_{xx})_{\max}] = \tau_C/\sigma_C$ (Fig. 9).

It is noted that both pure mode I and pure mode III cases are included in Figs. 4 and 7 as two special cases. Under pure mode I conditions, $K_I/K_{III} \rightarrow \infty$, Figs. 4 and 5 predict that tensile failure would happen

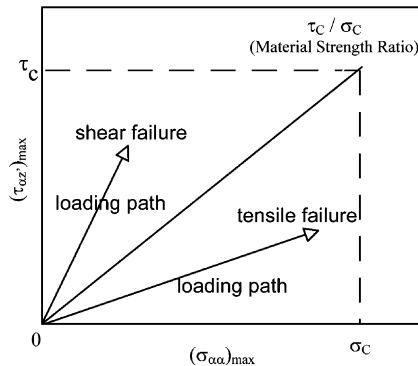


Fig. 6. Type of failure determined by $\tau_{\max}/\sigma_{\max}$.

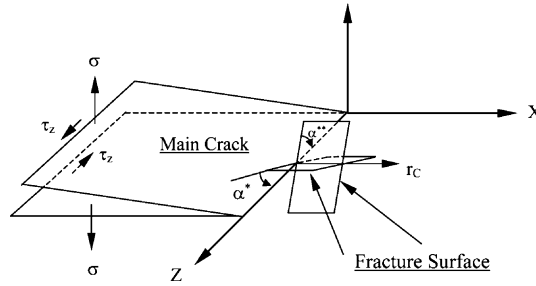


Fig. 7. Fracture surface or path at the crack tip; α^{**} (or α^*) corresponds to shear (or tensile) failure.

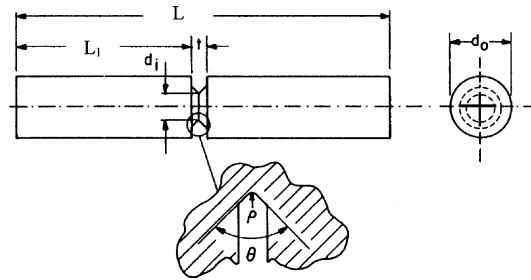


Fig. 8. Configuration of tension–torsion fracture specimen.

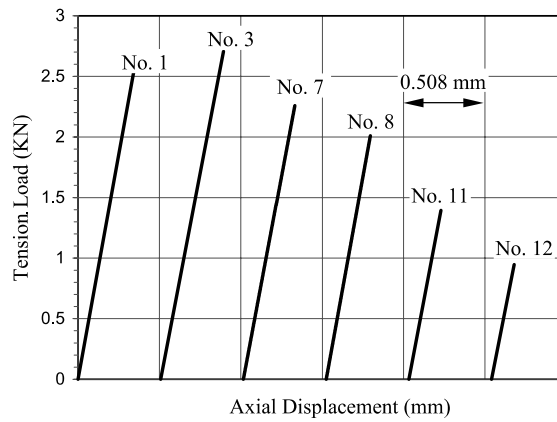


Fig. 9. Tension Load–axial displacement records for PMMA (Specimen No. 1 is under pure mode I loading).

for all ductile (τ_c/σ_c around 0.577) and brittle ($\tau_c/\sigma_c \geq 1$) materials. Under pure mode III conditions, $K_I/K_{III} = 0$ and the transition of fracture mechanism from tensile to shear depends upon the strength ratio of the material. The critical point is $\tau_c/\sigma_c = 1$ and a tensile fracture is predicted to occur for materials having the strength ratio $\tau_c/\sigma_c > 1$ and a shear fracture is predicted for materials having the strength ratio $\tau_c/\sigma_c < 1$. Figs. 4 and 7 also demonstrate that shear failure would not occur for materials having

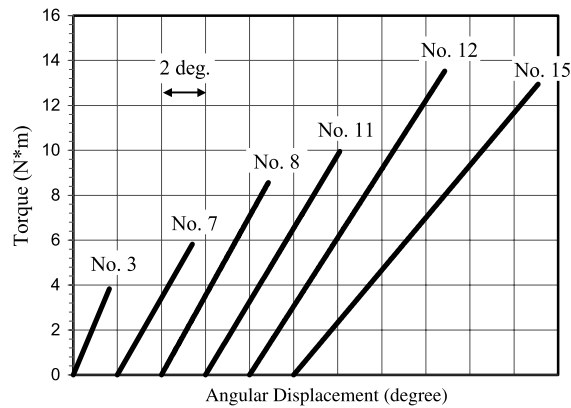


Fig. 10. Torque–Angular displacement records for PMMA (Specimen No. 15 is under pure mode III loading).

$\tau_C/\sigma_C > 1$ and tensile failure would not occur for materials having $\tau_C/\sigma_C < 0.167$ under plane strain conditions with $\nu = 1/3$ or $\tau_C/\sigma_C < 0.5$ under plane stress conditions (Fig. 10).

It is interesting to note that ductile materials with a strength ratio $\tau_C/\sigma_C = 0.5$ would fail in shear mode under plane stress conditions for any mode mixity K_I/K_{III} from Fig. 5. However, the same ductile materials, would fail in either tensile or shear mode under plane strain condition depending on the mode mixity K_I/K_{III} as shown in Fig. 4. Under mode I loading conditions, a ductile material having τ_C/σ_C close to 0.5 would fail in tensile type in plane strain and shear type in plane stress conditions. This might help explain the well-known shear lip formation in mode I fracture test of ductile materials, i.e., tensile failure in the center of the specimen and shear failure near the specimen surfaces as shear lips.

2.6. Direction of crack propagation

As discussed, meeting either a critical maximum tensile stress or a critical maximum shear stress could trigger a fracture event. Since the maximum tensile stress and the maximum shear stress is assumed to occur at the same radius around the crack tip whereas in two different orientations/locations, therefore the fracture plane or path would follow the fracture type, depending on the mode mixity and material strength ratio, and in the direction of either α^* from Eq. (16) or α^{**} from Eq. (22). Fig. 7 demonstrates the two potential fracture planes as determined by angles, α^* and α^{**} , in reference to the directions of $\sigma_{\theta\theta}|_{\theta=0}$, $\sigma_{xx}(\theta = 0, \alpha^{**})$ and $\tau_{xz'}(\theta = 0, \alpha^{**})$.

3. Experimental studies

3.1. Material properties

Two materials, PMMA and 7050 aluminum alloy, were tested in this work. They were chosen because they represent a brittle material and a ductile material, respectively. Material properties obtained from uniaxial tension tests at room temperature yield:

- (1) PMMA, elastic modulus = 2.95 GPa, yield strength (0.2% offset) = 44.60 MPa (Broviak, 1997).
- (2) 7050 aluminum alloy, elastic modulus = 69.51 GPa, yield strength (0.2% offset) = 379.21 MPa, ultimate tensile strength = 544.00 MPa.

3.2. Specimen geometry and fatigue pre-cracking

Circumferentially notched cylindrical bars were used for the tension–torsion of the mixed-mode I/III fracture tests. The principal objective of this work is to determine the fracture load and fracture path of notched rods containing “sharp” concentric flaws. A schematic diagram of the specimen is given in Fig. 8. The specimen have outside diameter $d_o = 15.24$ mm (0.6 in), inside diameter $d_i = 12.70$ mm (0.5 in), length of the rod $L = 152.40$ mm (6 in), $L_1 = 73.66$ mm (2.9 in), notch width $t = 5.08$ mm (0.2 in), notch angle $\theta = 60^\circ$, and notch radius $\rho = 0.152$ mm (0.006 in). The notch was introduced using a diamond wheel. Note that similar design of the test samples was used previously by Suresh and Tschegg (1987) and Tschegg and Suresh (1988) for ceramics, a relatively brittle material. This test specimen geometry enables an unequivocal use of linear elastic fracture mechanics to characterize the material toughness.

The notched rods were pre-cracked in uniaxial cyclic-tension to introduce a concentric fatigue crack. The growth of a uniform concentric fatigue pre-crack from the root of the notch was verified a posteriori through optical and scanning electron microscopy observations made after fracturing the specimens. The tension loading conditions were designed such that the maximum length of the concentric fatigue crack was almost the same for all specimens. The length of fatigue pre-crack was found to be 0.05–0.4 mm in both PMMA and aluminum alloy specimens which is intentionally kept small to minimize the crack-face frictional sliding which may elevate the mixed-mode fracture toughness.

3.3. Fracture testing

After introducing the fatigue pre-cracks, the specimens were fractured in various combinations of tensile and torsion loads using a specially designed grip on a MTS tension–torsion biaxial material testing machine. The equipment is designed such that the MTS machine holds the proper tension-torsion ratio during testing. Loading ramp rate is 6.35×10^{-3} mm/s (2.5×10^{-4} in/s). A program on a personal computer maintains the required ratio of the tension load P with respect to the torque T . Two specimens were tested for each of the seven P/T ratios ($N/(N * m)$): infinity (pure mode I), 954.094, 442.795, 255.669, 147.520, 68.504, 0.0 (pure mode III). Axial loads, torques and angular displacements were recorded and provided by the machine during the tests.

3.4. Test results of axial loads, torques and angular displacements

The specimens were loaded to failure in pure tension, combined tension and torsion, and pure torsion to investigate the effects of K_I , K_I and K_{III} , and K_{III} . Since K_I is dependent on the tensile load P only and K_{III} is dependent on the torque T only, the ratio of K_I/K_{III} was controlled by the proper ratio of P/T . The critical mode I and mode III stress intensity factor for fracture initiation was calculated using the following formulae (Suresh and Tschegg, 1987),

$$K_I = \frac{2P}{\pi d_i^2} \left[\frac{\pi d_i}{2} (1 - D) \right]^{\frac{1}{2}} (1 + 0.5D + 0.375D^2 - 0.363D^3 + 0.731D^4) \quad (29)$$

$$K_{III} = \frac{6T}{\pi d_i^3} \left[\frac{\pi d_i}{2} (1 - D) \right]^{\frac{1}{2}} (1 + 0.5D + 0.375D^2 + 0.3125D^3 + 0.273D^4 + 0.208D^5) \quad (30)$$

where $D = d_i/d_o$, P is the far field tension load and T is the far field applied torque at the onset of catastrophic fracture. Results of fracture toughness K_I and K_{III} are shown in Tables 1 and 2.

Figs. 11–14 show the tensile load versus axial displacement and torque versus angular displacement or angle of rotation for various K_I/K_{III} ratios for the two materials. Six curves are shown in the tensile load

Table 1

Test results and analysis of 7050 aluminum alloy ($\nu = 0.33$, from Metal Handbook, 1990)

No.	$\beta_{\text{eq-13}}$ (Degree)	K_I/K_{III} (MPa $\sqrt{\text{m}}$ /MPa $\sqrt{\text{m}}$)	α^* (MNSC) (Degree)	α^{**} (MSSC) (Degree)	$\tau_{\text{max}}\sqrt{r_C}/\sigma_{\text{max}}\sqrt{r_C}$ (MPa $\sqrt{\text{m}}$ /MPa $\sqrt{\text{m}}$)	α (exp) (Degree)
1	90	25.830/0 = ∞	0	45	1.752/10.305 = 0.170	0
2	90	26.120/0 = ∞	0	45	1.771/10.42 = 0.170	0
3	74.21	23.240/6.574 = 3.535	-29.50	15.50	3.059/10.754 = 0.284	-22.9
4	59.14	14.840/8.866 = 1.674	-37.06	7.94	3.677/8.591 = 0.428	-24.2
5	44.64	9.301/9.420 = 0.987	-40.24	4.76	3.811/6.890 = 0.553	-32.2
6	30.63	5.690/9.609 = 0.592	-42.13	2.87	3.853/5.737 = 0.672	11.6
7	31.81	5.959/9.609 = 0.620	-41.99	3.01	3.855/5.828 = 0.661	18.2
8	15.63	2.671/9.546 = 0.280	-43.64	1.36	3.813/4.697 = 0.812	0
9	15.96	2.579/9.017 = 0.286	-43.61	1.39	3.602/4.455 = 0.809	0
10	16.58	2.883/9.684 = 0.298	-43.55	1.45	3.868/4.823 = 0.802	0
11	0	0/9.646 = 0	-45	0	3.848/3.849 = 1.0	0

Table 2

Test results and analysis of PMMA ($\nu = 0.34$, from Broviak, 1997)

No.	$\beta_{\text{eq-13}}$ (Degree)	K_I/K_{III} (MPa $\sqrt{\text{m}}$ /MPa $\sqrt{\text{m}}$)	α^* (MNSC) (Degree)	α^{**} (MSSC) (Degree)	$\tau_{\text{max}}\sqrt{r_C}/\sigma_{\text{max}}\sqrt{r_C}$ (MPa $\sqrt{\text{m}}$ /MPa $\sqrt{\text{m}}$)	α (exp) (Degree)
1	90	1.048/0 = ∞	0	45	0.067/0.418 = 0.160	0
2	68.74	1.200/0.467 = 2.570	-33.83	11.17	0.201/0.604 = 0.333	-15.5
3	72.67	1.121/0.3498 = 3.205	-31.43	13.57	0.157/0.533 = 0.295	-13.8
4	72.71	0.817/0.254 = 3.212	-31.40	13.60	0.114/0.388 = 0.294	-14
5	72.67	1.107/0.346 = 3.204	-31.43	13.57	0.155/0.526 = 0.295	-13.5
6	57.52	0.931/0.593 = 1.571	-37.94	7.06	0.244/0.556 = 0.439	-24
7	57.82	0.935/0.588 = 1.589	-37.87	7.13	0.242/0.555 = 0.436	-28.3
8	42.24	0.830/0.914 = 0.908	-40.87	4.13	0.368/0.647 = 0.569	-32
9	44.12	0.827/0.853 = 0.970	-40.59	4.41	0.344/0.622 = 0.553	-35.5
10	29.00	0.627/1.131 = 0.554	-42.47	2.53	0.453/0.663 = 0.683	-42
11	29.78	0.577/1.009 = 0.572	-42.38	2.62	0.404/0.598 = 0.676	-42
12	14.98	0.391/1.460 = 0.268	-43.77	1.23	0.583/0.714 = 0.817	-43.5
13	14.57	0.412/1.586 = 0.260	-43.81	1.19	0.634/0.771 = 0.822	-43.2
14	0.09	0.003/1.747 = 0.002	-44.99	0.01	0.697/0.698 = 0.999	-45
15	0.39	0.010/1.456 = 0.007	-44.97	0.03	0.581/0.584 = 0.995	-45

versus axial displacement figures corresponding to P/T ratios $N/(N * m)$ of infinity (pure mode I), 954.094, 442.795, 255.669, 147.520, 68.504, and the torque versus angle of rotation figures corresponding to 954.094, 442.795, 255.669, 147.520, 68.504, 0.0 (pure mode III). Curves for PMMA specimen do not exhibit any non-linearity for any K_I/K_{III} ratio while curves for 7050 aluminum show obvious non-linear behavior, indicating certain plastic flow took place prior to fracture.

Fig. 13 shows the broken samples of the PMMA specimens subjected to various loading conditions of K_I/K_{III} . It clearly shows the increases of fracture angle or path from 0° to 45° as the K_I/K_{III} ratio decreases from ∞ (pure mode I) to 0.0 (pure mode III). Fracture angles were measured and listed in Table 2 as α° .

The fracture surface of 7050 aluminum alloy was flattened with shear rubbing marks. The fracture angles shown in Fig. 14 were measured by a special projection optical microscope and are then listed in Table 1. Note that a radial “factory roof” type fracture pattern is clearly shown in Fig. 14(a) and (b).

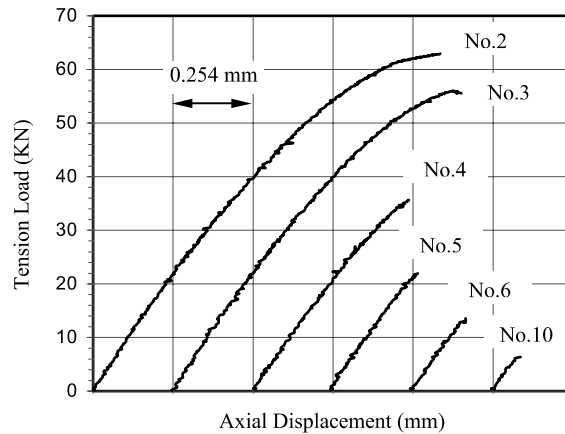


Fig. 11. Tension load–axial displacement records for 7050 aluminum alloy (Specimen No. 2 is under pure mode I loading).

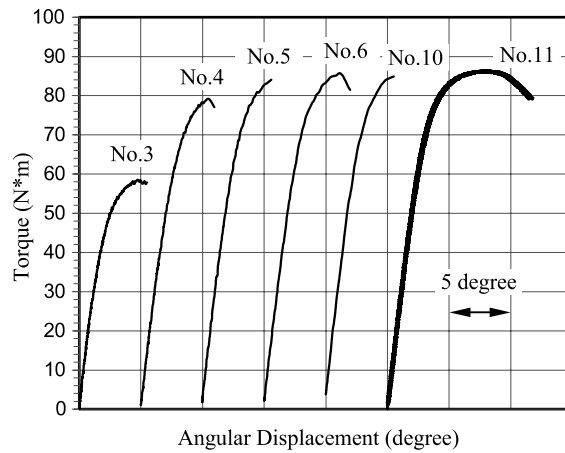


Fig. 12. Torque–angular displacement records for 7050 aluminum alloy (Specimen No. 11 is under pure mode III loading).

4. Analysis and discussions of experimental data

Table 1 lists the incipient crack propagation directions or kinking angle α (exp) from experiments, predicted angle α^* from MNSC using Eq. (16) and α^{**} from MSSC using Eq. (22), K_I and K_{III} corresponding to the fracture loads, and $\tau_{\max}/\sigma_{\max}$ calculated at the fracturing loads using Eqs. (18) and (24) for the specimens of 7050 aluminum alloy specimens. The notation $\tau_{\max}/\sigma_{\max}$ will be used in Tables 1 and 2, all Figures and discussions to replace $[(\tau_{\alpha\alpha'})_{\max}/(\sigma_{\alpha\alpha})_{\max}]$ for convenience. In addition, the loading mode mixity is represented by the equivalent crack angle

$$\beta_{\text{eq-13}} = \tan^{-1} \left(\frac{K_I}{K_{III}} \right) \quad (31)$$

to quantify the relative amount of mode I to mode III loading. As can be seen $\beta_{\text{eq-13}}$ ranges from $\pi/2$ to zero, with $\beta_{\text{eq-13}} = \pi/2$ corresponding to pure mode I and $\beta_{\text{eq-13}} = 0$ pure mode III. For the

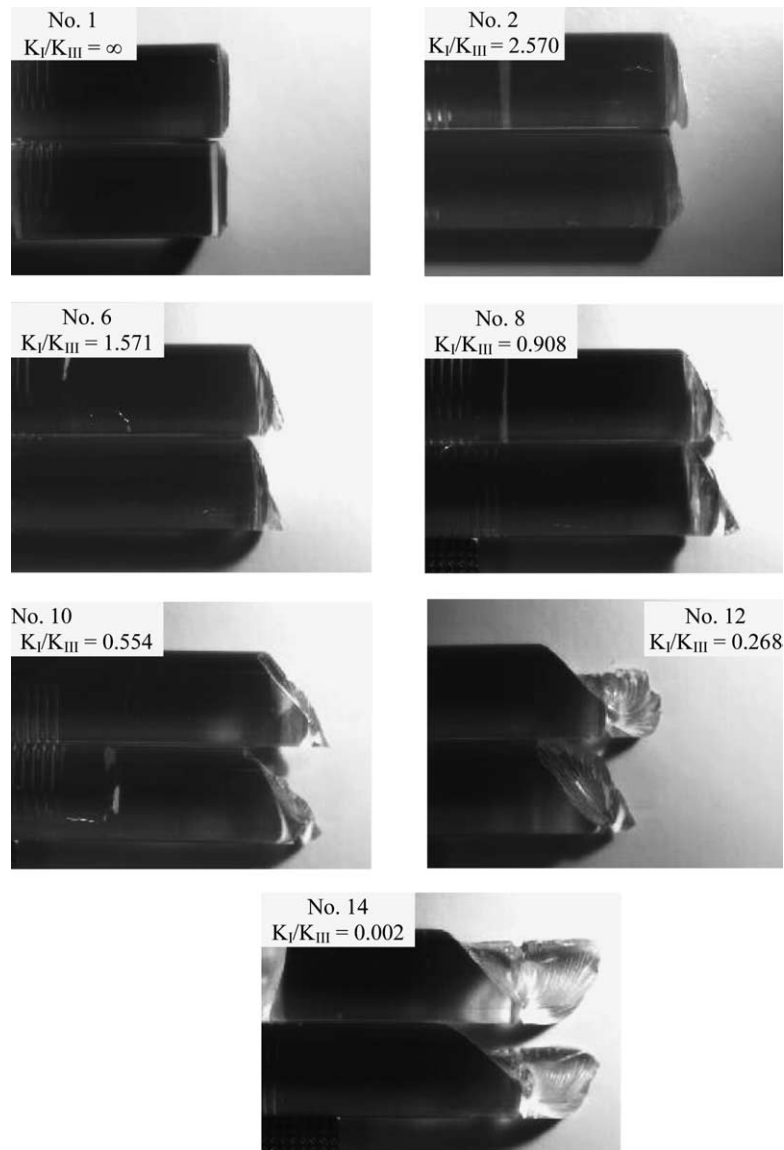


Fig. 13. Optical photograph of fractured PMMA specimens.

experimental data shown in Table 1, it should be noted that there are two types of fracture, i.e., tensile type for specimens No. 1–5 and shear type for specimens No. 6–11. Further discussions are given in the next section.

Table 2 shows similar test results for PMMA. In addition to our own test data, those of 4340 steel from Shah (1974), ceramics from Suresh and Tschegg (1987), and 26NCDV steel and TA5E Titanium under cyclic mixed mode loading (fatigue) conditions from Pook (1985) will also be included in our analysis. Note that except our current test data and Pook's test data, all data from other authors did not always report the observed fracture angles α and therefore some of them are missing in the following figures.

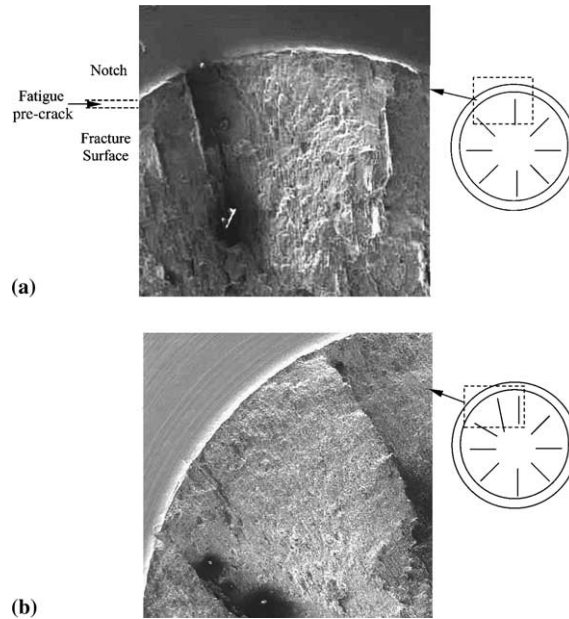


Fig. 14. Fractograph from scanning electron microscope showing tensile type of “factory roof” failure (a) (7050 aluminum alloy specimen, No. 4, $K_I/K_{III} = 1.674$, X20), (b) (7050 aluminum alloy specimen, No. 5, $K_I/K_{III} = 0.987$, X20).

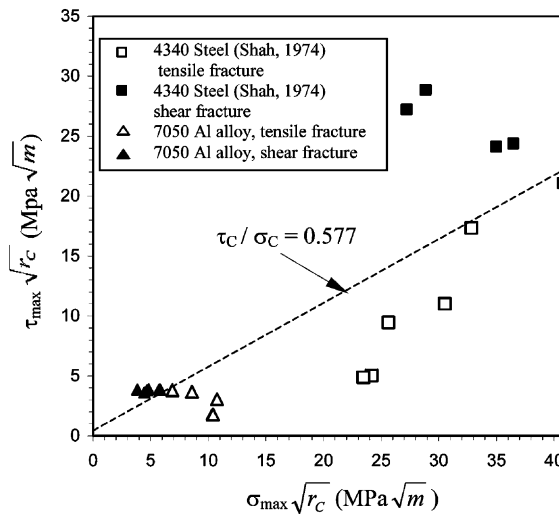


Fig. 15. Type of failure determined by τ_{max}/σ_{max} with experimental data (ductile materials).

4.1. Ductile materials – 7050 aluminum alloy and 4043 steel (Shah, 1974)

Shah (1974) examined the post fractured surface of 4340 steel and reported two types of fracture patterns, i.e., tensile fracture for some specimens and shear fracture for the rest of specimens. Since both our

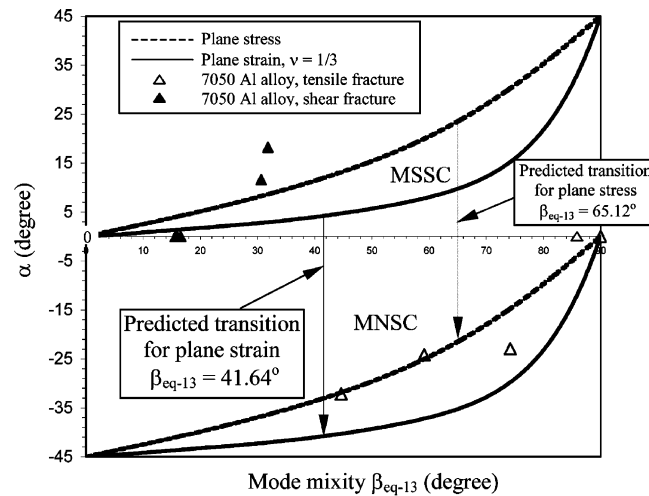


Fig. 16. Fracture initiation angle and mode transition; theoretical prediction and the experimental data.

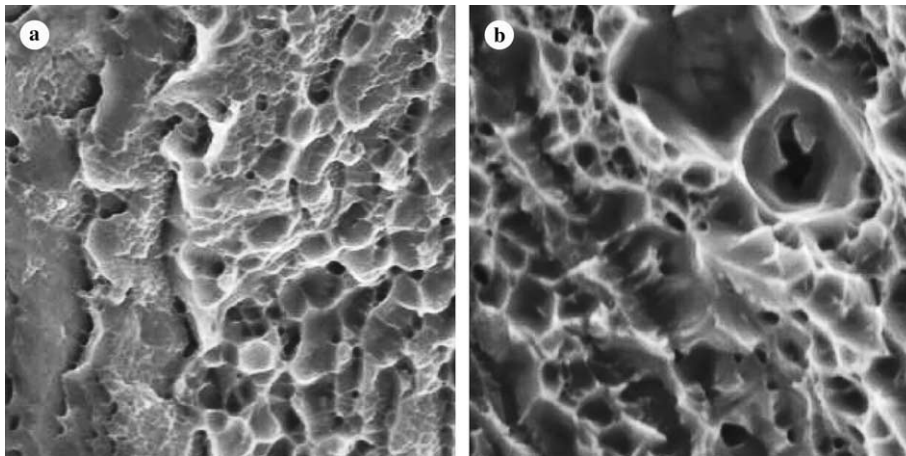


Fig. 17. Fractograph from scanning electron microscope showing tensile type of failure (a) (7050 aluminum alloy, No.1, $K_I/K_{III} = \infty$, X2000), (b) (7050 aluminum alloy, No. 5, $K_I/K_{III} = 0.987$, X2000).

7050 aluminum alloy and the 4340 steel by Shah (1974) failed in a ductile manner, the data in Table 1 and the data from Shah (1974) are plotted together in Fig. 15 as well as the line $\tau_C/\sigma_C = 0.577$ corresponding to von Mises failure theory. In Fig. 15, solid (hollow) symbols represent the shear (tensile) fracture as observed from the tests. It is seen from the figure that the type of fracture for all test data are predicted remarkably well by the theory, i.e., all test data below (above) the theoretical curve exhibit tensile (shear) fracture.

Fig. 16 shows the fracture initiation angles from both test data and theoretical predictions based on MNSC and MSSC versus the mode mixity β_{eq-13} . The predicted transition from tensile to shear type of fracture is at $\beta_{eq-13} = 41.64^\circ$ for plane strain conditions using Poisson ratio $\nu = 1/3$, a critical material strength ratio $(\tau_C/\sigma_C) = 0.577$ and Eqs. (15), (16), (19), (20), (22), (27) and (28). Again, all test data from ductile materials in Table 1, coincide well with the prediction. It should be mentioned that although the fracture angle was not reported, Shah (1974) stated that the transition of fracture type happened at an angle

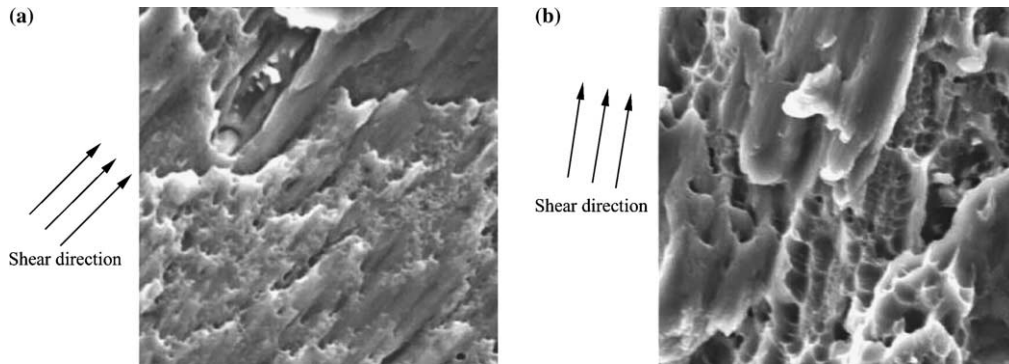


Fig. 18. Fractograph from scanning electron microscope showing shear type of failure (a) (7050 aluminum alloy, No. 6, $K_I/K_{III} = 0.592$, X2000), (b) (7050 aluminum alloy, No. 11, $K_I/K_{III} = 0.0$, X2000).

between $\beta_{eq-13} = 49.23^\circ$ ($K_I/K_{III} = 1.16$) and $\beta_{eq-13} = 32.25^\circ$ ($K_I/K_{III} = 0.631$) by examining the fractured surface. Our theoretical prediction of 41.64° falls in the middle of these two angles.

To further confirm the type of fracture, we examined the fracture surface of the broken specimens of aluminum 7050 under a scanning electronic microscope (SEM). The SEM observation from the region near the fatigue precrack tip reveals two distinct features. Group 1 specimens, i.e., No. 1–5 listed in Table 1, have equiaxed dimple patterns. Fractographs from specimen No. 1 and No. 5 are shown in Fig. 17(a) and (b). Group 2 specimens, consisting the rest of the specimens in Table 4, have elongated or parabolic dimple patterns, as shown in Fig. 18(a) and (b) from specimen No. 6 and No. 11. These patterns are typical and indicate: (a) the fracture is ductile and by microvoid growth and coalescence, and (b) group 1 specimens have tensile dominated failure and group 2 specimens have shear dominated failure. The fracture surface was flat with shear rubbing marks as the crack continues to grow deeper. These two types of fracture patterns are similar to those observed by Shah (1974) from 4340 steel and by Feng et al. (1993) from 2034 aluminum alloy.

In Fig. 18 the effect of stress state (plane stress or plane strain) on the fracture initiation angle as well as the predicted transition angle is also included. It is found that the predicted fracture initiation angles for plane stress conditions are smaller (larger) than those of plane strain conditions based on MNSC (MSSC) in the mixed mode regions. However, under pure mode I or pure mode III loading conditions, the predicted initiation angles for both the plane stress and plane strain conditions coincide. The difference shown in

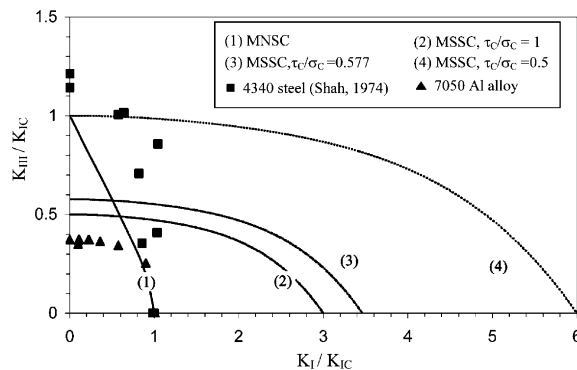


Fig. 19. Failure locus; theoretical prediction and experimental data ($n = 1$).

Fig. 18 for the two stress states suggests that the predicted crack propagation angle may be sensitive to the thickness of the specimens. Our tests used a round bar geometry and therefore at the crack tip it is believed to be close to the plane strain conditions.

Note that both 7050 aluminum alloy and 4340 steel are relatively ductile and have certain degrees of plasticity at the time of fracture. During fracture, the plastic zone sizes (in the direction of the crack plane) under pure mode I and mode III loading conditions as estimated by the Irwin's approach (Anderson, 1995) are

$$r_{\text{pI}} = \frac{1}{6\pi} \left(\frac{K_{\text{IC}}}{\sigma_s} \right)^2 \quad (32)$$

$$r_{\text{pIII}} = \frac{3}{2\pi} \left(\frac{K_{\text{IIIC}}}{\sigma_s} \right)^2 \quad (33)$$

where r_{pI} and r_{pIII} are the plastic zone size under mode I and mode III conditions, respectively; σ_s is the 0.2% offset yield strength. Substituting the values of $\sigma_s = 379.21$ MPa, $K_{\text{IC}} = 25.830$ and $K_{\text{IIIC}} = 9.646$ MPa m^{1/2} (Table 1) for 7050 aluminum alloy and $\sigma_s = 1475$ MPa, $K_{\text{IC}} = 60.588$ and $K_{\text{IIIC}} = 68.235$ MPa m^{1/2} (Shah, 1974) for 4340 steel into Eqs. (32) and (33), one obtains the plastic zone sizes under plane strain conditions as listed in Table 3. Also shown in Table 3 are the crack length as well as the inside radius ($d_i/2$) of the cracked round bar samples.

As can be seen from Table 3 comparatively, the plastic zone sizes (r_{pI} and r_{pIII}) are much smaller than the relevant dimensions of the sample. Small scale yielding assumption is therefore reasonable for current study. The prediction shown in Figs. 17 and 18 are however from linear elastic fracture mechanics. The effect of plasticity at the crack tip on the fracture initiation angle is not investigated due to the reason cited in Section 2. Nevertheless, it appears that the prediction is reasonably well. It is noted that Chao and Liu (1997) have shown that in the mode I/II problem the difference in the fracture propagation angle between the elastic and elastic-plastic cases is insignificant. And, this appears also to be the case here for the mode I/III problem.

Fig. 19 shows the loading level, K_I and K_{III} , from Table 1 and Shah (1974) as well as the theoretical fracture loci from the elastic case, Eq. (19) for the MNSC and Eq. (27) for the MSSC. K_I determined from pure mode I conditions, e.g. specimen No. 1 and No. 2 in Table 1, are used as K_{IC} in the Figures. It appears that the combination of MHSC and MSSC and including the transition, i.e. curves (1)–(3), can qualitatively predict the trend of the test data, as shown in Fig. 19. The experimental data near mode I region match well with the theoretical prediction (curve (1) from MHSC), while the experimental values of 7075 aluminum alloy (4340 steel) are rather lower (significantly higher) than the theoretical predictions (curves (2) and (3) from MSSC).

In summary, the comparison shown in Figs. 17, 18 and 21 indicate: (a) the fracture type and transition can be predicted very well, (b) the prediction for the fracture angle is reasonably well, and (c) only the trend of the fracture load locus can be predicted by the theory. The beauty of our current model is, by simply taking the strength ratio (τ_c/σ_c) at the same radius in front of crack tip, the requirement of knowing the exact value of the critical distance r_c as well as τ_c and σ_c is avoided. Yet, the crack initiation angles, the transition of fracture types and even the trend of fracture loci can still be predicted well.

Table 3
Plastic zone sizes and relevant dimensions of samples

Materials	r_{pI} (mm)	r_{pIII} (mm)	Crack length (mm)	$d_i/2$ (mm)
7050 Al	0.246	0.309	1.27	6.35
4340 Steel	0.090	1.022	4.83	6.60

There are several potential reasons attributed to the poor comparison in Fig. 19. Firstly, all test data shown in Table 1 and from Shah (1974) failed in a ductile manner. The local crack tip plasticity is not considered in the fracture loci from theory, however. Secondly, it is well known that the ductile fracture initiation is difficult to define and detect, i.e., it does not seem possible to measure where on the records the actual fracture begin and at what place in front of the crack edge. Therefore, the reported fracturing loads at the incipient of fracture may be in error. In comparison, the crack initiation angle, on the other hand, can normally be gauged precisely from the broken specimens after the tests. This could explain why the initiation angle, as shown in Fig. 18, is predicted well by the model. Thirdly, the friction of the crack surfaces would significantly contribute to the increase of the fracture load as the mixed mode I/III loading is applied, e.g., 4340 steel data points in Fig. 21. However, this friction effect is not considered in our model.

4.2. Brittle materials—PMMA and ceramics (Suresh and Tschegg, 1987)

Next, let us examine the test data shown in Table 2 from PMMA, and data from ceramics by Suresh and Tschegg (1987). Since PMMA and ceramic are rather brittle materials, we assume that they obey the maximum normal stress failure criterion, i.e., $(\tau_c/\sigma_c) = 1$. Plotted in Fig. 22 are the test data as well as the material strength ratio $(\tau_c/\sigma_c) = 1$. It is shown that all the test specimens are predicted to fail by tensile type of fracture that is confirmed by the test results. It appears that, for cracks loaded under mixed mode I/III and positive K_I , failure will *always* be the tensile type if the material obeys $(\tau_c/\sigma_c) = 1$ since the stress state $(\tau_{\max}/\sigma_{\max})$ is always less than unity for any mode mixity between, and including, pure mode I and pure mode III.

The experimentally observed crack propagation angles, shown in Table 2 for the brittle material PMMA, are plotted in Fig. 21 along with the theoretical prediction. Fig. 21 shows that: (a) all the experimental data follow the prediction from the MNSC, (b) no transition from tensile to shear type failure occurred which is consistent with the results shown in Fig. 20, and (c) the experimental data are close to the prediction by plane stress near mode I and plane strain near mode III. This could be due to the three-dimensional stress state in front of the circumferential crack in the notched bar geometry in contrast to the pure plane stress or pure plane strain assumptions. A complete explanation for this is not clear and warrants further investigation.

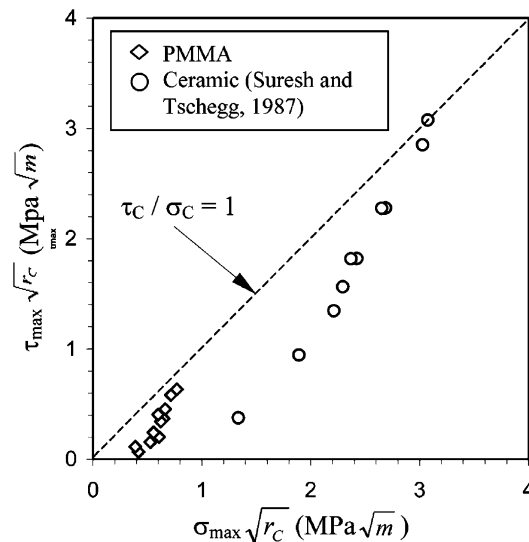


Fig. 20. Type of failure determined by $\tau_{\max}/\sigma_{\max}$ with experimental data; (PMMA and Ceramics, Suresh and Tschegg, 1987).

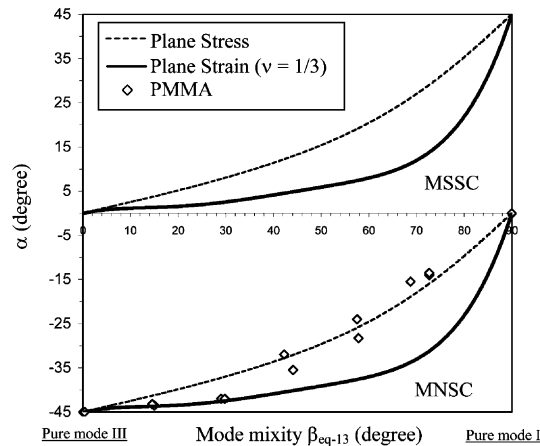


Fig. 21. Fracture angle; theoretical prediction and experimental data.

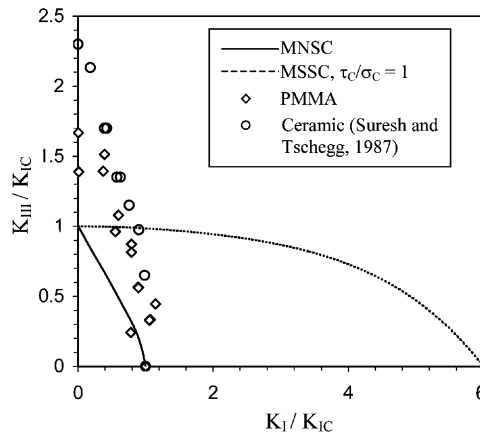


Fig. 22. Failure locus; theoretical prediction and experimental data; (brittle material—PMMA and ceramics, Suresh and Tschegg, 1987).

Fig. 22 shows the fracture loci with test data of PMMA from Table 2 and ceramics from Suresh and Tschegg (1987) along with the theoretical curves of Eq. (19) for the MNSC and (27) for the MSSC. Fig. 22 indicates that all the experimental data of PMMA and ceramics follow the trend predicted by the MNSC. The experimental data near mode I region match well with the theoretical prediction, while the values of the experimental data near mode III region are higher than the theoretical prediction. The elevation of toughness is apparently due to the crack surface friction, which is not included in our current model. Note that there is no intersection point from the two theoretical curves using $(\tau_c/\sigma_c) = 1$; thus no transition is predicted for materials having this strength ratio.

4.3. Fatigue test data—26NCDV steel and TA5E Titanium (Pook, 1985)

At last, it is anticipated that the proposed model in predicting the fracture path and types of fracture may also be applied to fatigue fracture. While the authors have not done a comprehensive literature review,

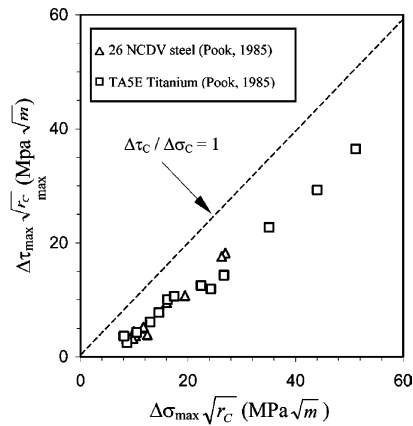


Fig. 23. Type of failure determined by $\Delta\tau_{\max}/\Delta\sigma_{\max}$ with experimental data; (fatigue test data of steel and Titanium, Pook, 1985).

the work by Pook (1985) offers an interesting trial case for extending the model to fatigue fracture. In his paper, discussing mixed mode I/III fatigue test, Pook (1985) reported fracture type and path of 26 NCDV rotor steel (3.5 Ni–Cr–Mo–V) and TA5E Titanium alloy (5 Al–2.5 Sn). Relevant data is shown in Fig. 23. The rotor steel has very high yield and tensile strength and the fatigue loads are relatively low. As such, the cyclic plastic deformation and the crack tip plastic zone size are very small. The Titanium has an ultimate tensile strength of 793–828 MPa and ultimate shear strength 759 MPa as listed in (Aerospace Structural Metals Handbook, 1993), 10th Edition, Vol. 4. These values give $(\tau_c/\sigma_c) = 0.92\text{--}0.96$ which is a rather “brittle” material. It is therefore assumed that the two materials obey the maximum normal stress failure criterion, i.e., $(\tau_c/\sigma_c) = 1$, in our interpretation. Plotted in Fig. 23 are the test data and the material strength ratio $(\Delta\tau_c/\Delta\sigma_c) = 1$, assuming that the material strength ratio can be applied to fatigue. As shown in Fig. 23 all test data points are below the material strength ratio line indicating the fracture should all follow MNSC prediction, or tensile type. This is indeed as observed and reported by Pook (1985).

The experimental crack initiation angles are plotted in Fig. 24 along with the theoretical prediction. It can be seen from Fig. 24 that all the observed fatigue fracture angles are close to, but smaller than, the

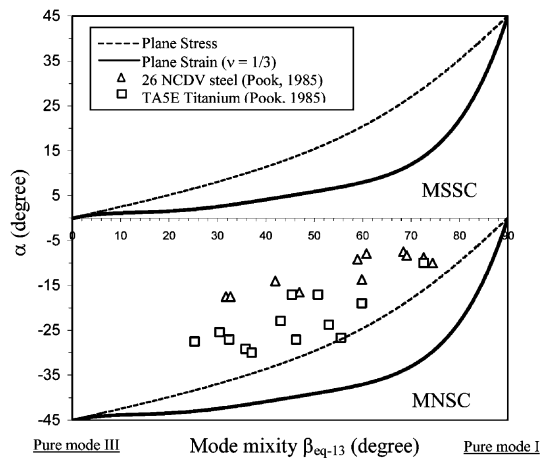


Fig. 24. Fracture angle; theoretical prediction and experimental data (fatigue testing of steel and Titanium Pook, 1985).

predicted initiation angles based on the MNSC indicating tensile fracture. No shear fracture is observed in the experiments as predicted. The discrepancy of the initiation angles between the theoretical prediction and experimental results could be due to the frictional contact between the crack surfaces, i.e., the initial “factory roof” facets may be formed and then rubbed upon, which prohibited the full development of the fracture initiation angles.

5. Conclusions

The present paper, attempts to bridge the failure properties of materials, as determined from the strength tests of smooth specimens, with the fracture behavior of solids containing a crack. Mixed mode I/III fracture is studied. Using the elastic crack tip stress field and the mathematic theorem of extreme values, the maximum normal stress and the maximum shear stress at an inclined plane with the same distance from the crack front are uniquely determined. A criterion based on the competition of these two maximum stresses and combined with the material failure strength ratio for the prediction of fracture type is developed. The proposal is validated by comparison to extensive experimental data from different materials. It is found that: (a) the elastic theory predicts the crack propagation direction and the tensile-shear transition accurately for both brittle and ductile engineering materials, (b) the fracture locus using the elastic theory is not sufficient and including plasticity and friction between crack surfaces could improve the quantitative prediction; and (c) the present theory could be extended to the interpretation of fracture type transition in fatigue tests of structural metals.

Note that using the same critical distance r_c for evaluating both the critical tensile and shear stress is an assumption. While no one really knows the accuracy of this assumption, the result in predicting the fracture type seems to be satisfactory. The same is true for using the elastic stress for the evaluation of the transition. Note that the “actual” value of the stress, either hoop or shear, is never used in the proposed theory for predicting the transition. Only the ratio of these two stresses is used for comparison with the material strength ratio. Under the proportional loading condition, the ratio of the stress components could maintain a nearly constant value from elastic to the fully plastic regime and this could attribute to the success of the prediction shown in the paper.

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